NASA CR-172,558

NASA Contractor Report 172558

ICASE REPORT NO. 85-14

NASA-CR-172558
19850069877

ICASE

INFORMATION CONTENT IN SPECTRAL CALCULATIONS

Saul Abarbanel
David Gottlieb

LIBRARY COPY

MAR 1985

LANGLEY RESEARCH CENTER
LIBRARY, NASA
HAMPTON, VIRGINIA

Contract No. NAS1-17070
February 1985

INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING
NASA Langley Research Center, Hampton, Virginia 23665

Operated by the Universities Space Research Association

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

INFORMATION CONTENT IN SPECTRAL CALCULATIONS

Saul Abarbanel
Tel-Aviv University
and
Institute for Computer Applications in Science and Engineering

David Gottlieb
Tel-Aviv University
and
Institute for Computer Applications in Science and Engineering

Abstract

Spectral solutions of hyperbolic partial differential equations induce a Gibbs phenomenon near local discontinuities or strong gradients.

A procedure is presented for extracting the piecewise smooth behavior of the solution out of the oscillatory numerical solution data. The procedure is developed from the theory of linear partial differential equations. Its application to a non-linear system (the two-dimensional Euler equations of gas dynamics) is shown to be efficacious for the particular situation.

To be published in the series "Progress in Scientific Computations," Birkhauser Inc., Boston, Proceedings of the Bi-National U.S./Israel Workshop on "Impact of Supercomputers on the Next Decade of CFD," Jerusalem, December 1984.

Research was supported in part by the Air Force Office of Scientific Research under Contract No. AFOSR 83-0089 and in part by the National Aeronautics and Space Administration under NASA Contract No. NAS1-17070 while the authors were in residence at ICASE, NASA Langley Research Center, Hampton, VA 23665.

N85-71896

1. INTRODUCTION

It is well known that when one solves numerically, with the aid of higher order approximations, partial differential equations or systems one may get numerical oscillations. The oscillations may arise from different sources, e.g. incorrect treatment of the outflow boundaries in hyperbolic systems, mild non-linear instabilities, etc. One interesting class of numerical oscillations occurs when flows with extreme gradients, or local discontinuities, are simulated. The usual way of combating this manifestation of the Gibbs phenomenon is to introduce some kind of artificial viscosity (even TVD methods basically do this).

When the basic algorithm used in the computation has spectral accuracy, the above strategy is not viable since it will reduce the order of accuracy drastically. Even hybrid "switches" do not overcome this difficulty.

The approach taken in this paper is based on the premise that, because of their very high accuracy, spectral methods, regardless of the smoothness or lack of it on the computational grid, retain all the information* in the form of the solution interpolant in a hyperplane whose projection we see on the grid points.

In Section 1 we develop the theory for the case of a linear hyperbolic operator with constant coefficients and periodic boundary conditions. A numerical example is then presented which demonstrates the efficacy of the proposed procedure for extracting the piecewise smooth solution from the oscillatory raw data.

In Section 2 an analogous procedure is developed for the non-periodic case where the interpolating polynomials are not trigonometric but

---

* Here by information we do not mean information in the formal way as defined by Shannon [2], but rather by following Lax [3] – the ability to use the data to construct a piecewise smooth looking solution on the grid points.

Tchebyshev. The application is then tested on a physical problem - two dimensional supersonic wedge flow. The raw data in this example is the density as obtained by a pseudospectral steady state solution of the 2-D Euler equations. The postprocessing procedure is again found to be effective, particularly with regard to the (2-D) shock location and the magnitude of the density jump.

2. FOURIER METHODS

Consider the following initial-boundary value problem

$$u_t + u_x = 0 \qquad 0 < x < 2\pi , \qquad t > 0 \tag{2.1}$$

$$u(x,0) = g(x) \qquad 0 \leq x \leq 2\pi \tag{2.2}$$

$$u(x,t) = u(x+2\pi,t) \tag{2.3}$$

where $g(x)$ is piecewise smooth and has a discontinuity at a point $x = \bar{x}$, say. When problem (2.1) - (2.3) is solved by the pseudospectral Fourier method, [1], the initial condition $g(x)$ is replaced by its N-th order trigonometric interpolant, $g_N(x)$:

$$g_N(x) = \sum_{\ell=-N}^{N} \hat{g}_\ell e^{i\ell x} \tag{2.4}$$

where

$$\hat{g}_\ell(x) = \frac{1}{2Nc_\ell} \sum_{j=0}^{2N-1} g(x_j) e^{-i\ell x_j} \tag{2.5}$$

$$x_j = \frac{\pi j}{N} \qquad j = 0,1,\ldots,2N-1 \tag{2.6}$$

and

$$c_\ell = \begin{cases} 1 & |\ell| < N \\ 2 & |\ell| = N . \end{cases} \tag{2.7}$$

Since (2.1) is linear with constant coefficients, the pseudospectral approximation, $u_N(x,t)$, satisfies the following problem

$$\frac{\partial u_N}{\partial t} + \frac{\partial u_N}{\partial x} = 0 \tag{2.8}$$

$$u_N(x,0) = g_N(x) \tag{2.9}$$

$$u_N(x,t) = u_N(x+2\pi,t) \ .$$

The solution to (2.8) is

$$u_N(x,t) = g_N(x-t) \ . \tag{2.10}$$

Since $g_N(x)$ collocates $g(x)$ at the points $0 \leq j \leq 2N-1$, the discrete values $g_N(x_j)$ convey visually the impression of a piecewise smooth function, which $g(x)$ indeed is. If however we examine $g_N(x)$ at points other than the collocation points x_j, the discrete values of $g_N(x)$ thus examined form an oscillatory pattern. Therefore if we evaluate $u_N(x_j,t) = g_N(x_j-t)$, at $t \neq \pi m/N$ (m an integer), then the solution (2.10) will indeed look oscillatory. If $g(x)$ were a smooth function, then these oscillations would be very small in magnitude and in fact would not be discerned by eye. The discontinuity at x_s, however, induces much larger oscillations, due to the Gibbs phenomenon. Note, however, that if $t = \pi m/N$ then the solution values at the collocation points are spectrally accurate. It is in this sense that we make the claim that the pseudospectral solution retains the same amount of information for problems with discontinuous initial data as for smooth initial conditions.

Our basic interest is in the "inverse problem": given the numerical results of a pseudospectral procedure applied to a hyperbolic problem (possibly even a non-linear one) which look oscillatory on the collocation points, $x_j = \pi j/N$ — how do we extract out of this data the correct piecewise smooth solution (i.e. the location of the discontinuity, the magnitude of the jump, and the structure of the smooth part of the solution)?

The approach we shall use to recover the "lost" information is to consider first (2.1) - (2.3) with $g(x)$ being the saw-tooth function:

$$g(x) = AF(x,x_k) = \begin{cases} Ax & x < x_k \\ A(x-2\pi) & x_k \leq x \ , \quad (k \text{ integer}) \end{cases} \tag{2.11}$$

where $2\pi A$, then, is the magnitude of the jump. We use (2.11) because this saw-tooth function is the simplest periodic piecewise smooth function with a given discontinuity. We call (2.1) - (2.3) with $g(x)$ given by (2.11) the fundamental problem. The pseudospectral approximation to the solution of the fundamental problem satisfies, by analogy to (2.8) - (2.9), the following set:

$$\frac{\partial v_N}{\partial t} + \frac{\partial v_N}{\partial x} = 0 \tag{2.12}$$

$$u_N(x,0) = AF_N(x,x_k) \tag{2.13}$$

with $F_N(x,x_k)$ given by:

$$F_N(x,x_k) = \sum_{\ell=-N}^{N} a_\ell(k) e^{i\ell x} , \tag{2.14}$$

$$a_\ell(k) = \frac{1}{2Nc_\ell} \sum_{j=0}^{2N-1} e^{-i\ell x_j} \cdot F_N(x_j,x_k) . \tag{2.15}$$

Performing (2.15) we get:

$$a_0(k) = \frac{\pi}{N} [k - N + .5] \tag{2.16}$$

$$a_\ell(k) = \frac{\pi}{2Nc_\ell} \left[2 \cdot \frac{1 - e^{-\frac{i\pi\ell(k+1)}{N}}}{1 - e^{-\frac{i\pi\ell}{N}}} + i \operatorname{ctn} \frac{\pi\ell}{2N} - 1 \right] \quad (\ell \neq 0) . \tag{2.17}$$

The solution to (2.12) - (2.13) is

$$\begin{aligned} v_N(x,t) &= AF_N(x-t,x_k) \\ &= A \sum_{\ell=0}^{2N-1} a_\ell(k) e^{i\ell(x-t)} \\ &= A \left\{ \sum_{\ell=0}^{2N-1} a_\ell(k+\theta) e^{i\ell x} - t \right\} \\ &= A[F_N(x,x_k+t) - t] \end{aligned} \tag{2.18}$$

where

$$\theta = \frac{Nt}{\pi} \quad . \tag{2.19}$$

The last two equalities in (2.18) are obtained by manipulating (2.14) - (2.17).

We now use the pseudospectral approximation to the fundamental problem, $v_N(x,t)$, in order to remove the oscillations in (2.10). In fact, $\varphi_N(x,t) = u_N(x,t) - AF_N(x)$ is a continuous function satisfying the following p.d.e. and initial data:

$$\frac{\partial \varphi_N}{\partial t} + \frac{\partial \varphi_N}{\partial x} = 0 \tag{2.20}$$

$$\varphi_N(x,0) = g_N(x) - AF_N(x). \tag{2.21}$$

We note that $\varphi_N(x,0)$ is a continuous function because $g_N(x) - AF_N(x)$ does not have a jump anymore. It follows that $\varphi_N(x,t)$ converges to $\varphi(x,t)$ faster than $u_N(x,t)$ does to $u(x,t)$.

The above observations lead to the following procedure that extracts from $u_N(x_j,t)$, which is an oscillatory approximation to $u(x,t)$, a piecewise smooth representation of $u(x,t)$. Given $u_N(x,t) = \sum_{\ell=-N}^{N} \hat{u}_\ell e^{i\ell x}$, we try to find an unknown smooth and a saw-tooth function $AF_N(x-t,x_s)$ with an unknown jump $2\pi A$ at an unknown location x_s , such that

$$H = \sum_{j=0}^{2N-1} \left[u_N(x_j,t) - AF_N(x_j,x_s) - c - \sum_{\substack{\ell=-p \\ \ell\neq 0}}^{p} b_\ell e^{i\ell x_j} \right]^2 \tag{2.22}$$

is minimized. In (2.22) c is an unknown constant which when solved for will yield $t = c/A$ (see 2.18); $|p| < N$ and b_0 is taken to be zero. The inner sum in (2.22) represents the continuous function we seek. Note that we have $2p + 3$ unknowns in (2.2): A, x_s, c, and $2p$ values of b_ℓ $(\ell\neq 0)$.

The conditions for local minima of H are found from the following $2p + 3$ equations:

$$\frac{\partial H}{\partial A} = 0 \Rightarrow \sum_{j=0}^{2N-1} \left[u_j F_j - AF_j^2 - cF_j - F_j \sum_{\substack{\ell=-p \\ \ell\neq 0}}^{p} b_\ell e^{i\ell x_j} \right] = 0 \tag{2.23}$$

where $F_j = F_N(x_j,x_s)$, $u_j = u_N(x_j,t)$. Also

$$\frac{\partial H}{\partial c} = 0 \Rightarrow \sum_{j=0}^{2N-1} \left[u_j - AF_j - c - \sum_{\substack{\ell=-p \\ \ell\neq 0}}^{p} b_\ell e^{i\ell x_j} \right] = 0 \tag{2.24}$$

$$\frac{\partial H}{\partial s} = 0 \Rightarrow \sum_{j=0}^{2N-1} \left[F_j' u_j - AF_j' F_j - cF_j' - F_j' \sum_{\substack{\ell=-p \\ \ell\neq 0}}^{p} b_\ell e^{ix_j\ell} \right] = 0 \tag{2.25}$$

where $F_j' = \partial F_N(x_j, x_s)/\partial s = \sum_{\ell=-N}^{N} \frac{\partial a_\ell(s)}{\partial s} \cdot e^{i\ell x_j}$;

and

$$\frac{\partial H}{\partial b_m} = 0 \Rightarrow b_m = \hat{u}_m - Aa_m \ , \quad |m| = 1,2,\ldots,p \tag{2.26}$$

where $\hat{u}_m = \frac{1}{2Nc_m} \sum_{j=0}^{2N-1} u_N(x_j) e^{-i\ell x_j}$.

Substituting (2.26) into (2.23), (2.24), and (2.25) we get, respectively:

$$\hat{u}_0 - Aa_0 - c = 0 \tag{2.27}$$

$$\sum_{|\ell|>p} c_\ell a_{-\ell} \hat{u}_\ell - A \sum_{|\ell|>p} c_\ell a_{-\ell} a_\ell = 0 \tag{2.28}$$

$$\sum_{|\ell|>p} c_\ell a'_{-\ell} \hat{u}_\ell - A \sum_{|\ell|>p} c_\ell a'_{-\ell} a_\ell = 0 \tag{2.29}$$

where $a'(s) = \partial a_\ell(s)/\partial s$. Next we combine (2.28) and (2.29) to get a single non-linear equation for s :

$$\sum c_\ell a'_{-\ell} \hat{u}_\ell \sum c_\ell a_{-\ell} a_\ell - \sum c_\ell a_{-\ell} \hat{u}_\ell \sum c_\ell a'_{-\ell} a_\ell = 0 \tag{2.30}$$

where all sums run over $p < |\ell| \leq N$.

Equation (2.30) is solved iteratively for s . Having found s , one immediately obtains from (2.16) and (2.17) all the $a_\ell(s)$'s. Then from (2.26) we have the b_m's , and A from (2.28). Finally, having A we find c from (2.27).

The minimum thus obtained may be a local one while we are seeking a global minimum. This means that in practice one searches for the global minimum.

We now give an example that illustrates the efficacy of the procedure. We solve the following problem:

$$\frac{\partial u_N}{\partial t} + \frac{\partial u_N}{\partial x} = 0 \qquad 0 < x < 2\pi \ , \quad t > 0 \tag{2.31}$$

$$u_N(x,0) = \begin{cases} \sin \frac{x}{2} & 0 \leq x \leq \pi \\ -\sin \frac{x}{2} & \pi \leq x \leq 2\pi \end{cases} \tag{2.32}$$

$$u_N(0,t) = u_N(2\pi,t) . \tag{2.33}$$

We ran the problem on several grids and exhibit here the numerical results for the case $N = 8$ (i.e., 16 subintervals in the domain $(0,2\pi)$). The unadulterated results at $t = \pi/2N$ on the grid points are shown in Figure 1, and the corresponding pointwise errors are listed in Table 1.

Table 1

j	exact solution	error 1 = |exact-unsmoothed|	error 2 = |exact-smoothed|	error 1 / error 2
0	9.80×10^{-2}	5.86×10^{-5}	5.86×10^{-5}	1.00
1	9.80×10^{-2}	1.24×10^{-2}	5.86×10^{-5}	211
2	2.90×10^{-1}	2.57×10^{-2}	6.30×10^{-5}	408
3	4.71×10^{-1}	4.13×10^{-2}	7.33×10^{-5}	563
4	6.34×10^{-1}	6.15×10^{-2}	9.30×10^{-5}	661
5	7.73×10^{-1}	9.11×10^{-2}	1.31×10^{-4}	695
6	8.82×10^{-1}	1.43×10^{-1}	2.16×10^{-4}	662
7	9.57×10^{-1}	2.70×10^{-1}	4.42×10^{-4}	611
8	-9.95×10^{-1}	1.00×10^{0}	1.10×10^{-2}	91
9	-9.95×10^{-1}	2.68×10^{-1}	1.34×10^{-3}	200
10	-9.57×10^{-1}	1.42×10^{-1}	4.42×10^{-4}	321
11	-8.82×10^{-1}	9.07×10^{-2}	2.16×10^{-4}	420
12	-7.73×10^{-1}	6.12×10^{-2}	1.32×10^{-4}	464
13	-6.34×10^{-1}	4.11×10^{-2}	9.30×10^{-5}	442
14	-4.71×10^{-1}	2.55×10^{-2}	7.32×10^{-5}	348
15	-2.90×10^{-1}	1.22×10^{-2}	6.30×10^{-5}	194

We then postprocessed these $u_N(x_j,\pi/2N)$ values according to the procedure described above. The filtered values are shown on the same graph, and the errors are listed in the same table as the unprocessed ones. The dramatic improvement is evident.

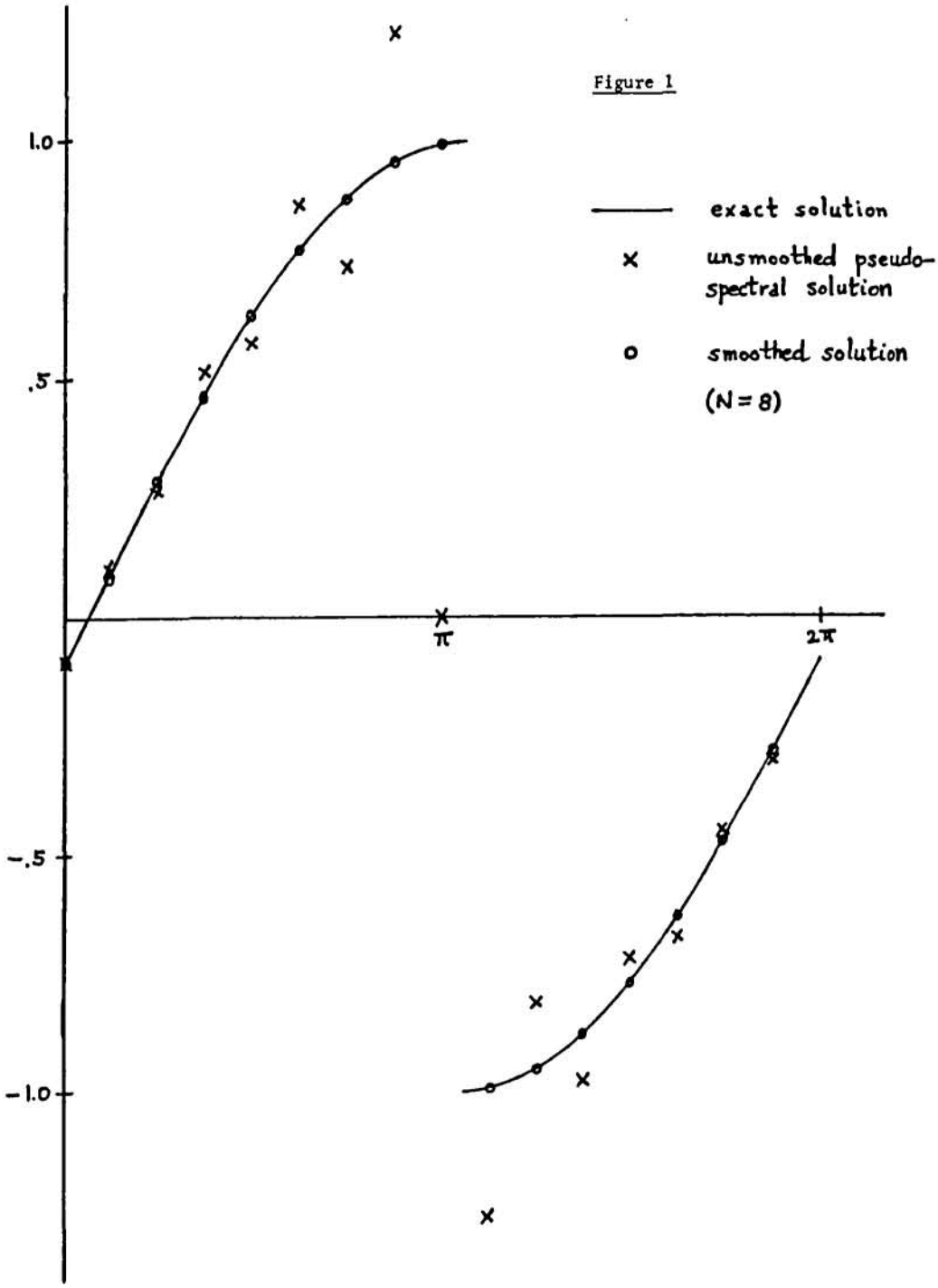
1.0
.5
-.5
-1.0
π
2π
exact solution
unsmoothed pseudo-spectral solution
smoothed solution
(N = 8)

Figure 1

In the next section we consider a two dimensional hyperbolic system of non-linear equation, namely the Euler equations. Because the physical problem involves inflow, outflow, and no-flow boundary conditions, periodicity could not be imposed; and we use the Tchebyshev, rather than Fourier, pseudospectral method.

3. TCHEBYSHEV METHODS — THE WEDGE FLOW PROBLEM

The physical problem is that of a wedge, inserted at a zero angle of attack, into a uniform supersonic flow of an ideal gas with $\gamma = 1.4$. An oblique shock develops in time, and the flow reaches after a while a steady state.

The time dependent Euler equations in two space dimensions were discretized by the pseudospectral Tchebyshev method in space with an 8×8 grid and a modified Euler scheme was used for the time discretization (see [4]). Since we are interested in the steady state only, the accuracy for the time integration is of little importance. In order to be sure that a steady state is reached, the code was run until all physical quantities did not change to 11 significant figures over a span of 100 time steps. The values of the density in the steady state at the grid points together with the grid points themselves are given in Table 2.

Table 2

ρ									Y
1.862	1.851	1.869	1.871	1.837	1.865	1.892	1.885	1.878	1.
1.862	1.870	1.867	1.820	1.870	1.954	1.899	1.803	1.759	.961
1.862	1.854	1.852	1.904	1.877	1.770	1.782	1.864	1.900	.853
1.862	1.871	1.876	1.812	1.838	1.969	1.975	1.884	1.841	.691
1.862	1.848	1.842	1.935	1.899	1.703	1.710	1.890	1.984	.5
1.862	1.883	1.894	1.729	1.832	2.429	2.994	3.255	3.316	.308
1.862	1.808	1.810	2.387	3.133	3.375	3.224	3.054	3.002	.146
1.862	2.115	2.868	3.288	3.176	2.965	3.006	3.136	3.187	.038
1.862	3.083	3.046	2.975	3.087	3.108	3.024	3.013	3.016	0
X 0	.038	.146	.308	.5	.691	.853	.961	1.	

Note that the raw data in Table 2 seem to indicate roughly the same y-shock location at $x_0 = 1$, $x_1 = .961$, and $x_2 = .853$, namely

between the grid points $y_4 = .3086$ and $y_5 = .500$. This means that because of the coarse Tchebyshev grid the shock location cannot be resolved to better than 20% of the domain. In fact the correct shock locations at those x-stations are $y = .434$ for x_0, $y = .417$ for x_1, and $y = .370$ for x_2.

In the present case it is not necessary to employ a saw-tooth piecewise smooth function, as was done in the previous section, because there is no need to preserve periodicity. Instead we substract from the oscillatory data an expansion of the Heaviside function, $S(y,y_s)$:

$$S(y,y_s) = \begin{cases} d_1 + d_2 & -1 \leq y \leq y_s \\ d_1 & y_s \leq y \leq 1 \end{cases} \tag{3.1}$$

where d_1, the state ahead of the shock, and d_2, the magnitude of the discontinuity, are constant. The description here of $S(y,y_s)$ as if independent of x has to do with the fact that the 2 dimensional results of the pseudospectral algorithm were post-processed at fixed x-stations. The expansion of $S(y,y_s)$ is given by

$$S_N(y,y_s) = \sum_{\ell=0}^{N} A_\ell(s)T_\ell(y) \tag{3.2}$$

where $T_\ell(y)$ is the Tchebyshev polynomial of order ℓ, $T_\ell(y) = \cos[\ell \cos^{-1}(y)]$, and

$$A_0(s) = (s + \tfrac{1}{2})/N$$

$$A_\ell(s) = \sin\left[\tfrac{\pi\ell}{N}(s + \tfrac{1}{2})\right]/N \sin\tfrac{\pi\ell}{2N} \quad ; \quad 1 \leq \ell \leq N-1$$

$$A_N(s) = \sin\left[(s + \tfrac{1}{2})\right]/2N.$$

If s is an integer then on the grid points, $y_j = \cos(\pi j/N)$,

$$S_N(y_j,y_s) = S(y_j,y_s). \tag{3.3}$$

The L_2-norm which we wish to minimize is now, at any given x-station:

$$H = \sum_{j=0}^{N} \frac{1}{c_j}\left[\rho_N(y_j) - d_1 - d_2 S_N(y_j,y_s) - \sum_{\ell=1}^{p<N} b_\ell T_\ell(y_j)\right]^2 \tag{3.4}$$

where

$$c_j = \begin{cases} 1 & 1 \leq j \leq N-1 \\ 2 & j = 0,\ N\ . \end{cases} \tag{3.5}$$

Differentiating (3.5) with respect to the parameters d_1 , d_2 , s and $b_\ell (1 \leq \ell \leq p < N)$, using the orthogonality relations for the Tchebyshev polynomials and manipulations similar to those used in the previous section, we get $p + 3$ nonlinear algebraic equations which are completely analogous to (2.26) - (2.29). They are:

$$b_\ell = \hat{\rho}_\ell - d_2 A_\ell\ , \qquad \ell = 1,\ 2,\ldots,\ p. \tag{3.6}$$

$$\hat{\rho}_0 - d_2 A_0 - d_1 = 0 \tag{3.7}$$

$$\sum_{\ell=p+1}^{N} c_\ell A_\ell \hat{\rho}_\ell - d_2 \sum_{\ell=p+1}^{N} c_\ell A_\ell^2 = 0 \tag{3.8}$$

$$\sum_{\ell=p+1}^{N} c_\ell A_\ell' \hat{\rho}_\ell - d_2 \sum_{\ell=p+1}^{N} c_\ell A_\ell A_\ell' = 0 \tag{3.9}$$

where

$$\hat{\rho}_\ell = \frac{2}{N c_\ell} \sum_{j=0}^{N} \frac{1}{c_j} \rho(y_j) T_\ell(y_j) \tag{3.10}$$

$$A' = \frac{\partial}{\partial s} A_\ell(s)\ . \tag{3.11}$$

Again, we combine (3.8) and (3.9) into a single nonlinear equation for the shock location index, s :

$$\sum c_\ell A_\ell' \hat{\rho}_\ell \sum c_\ell A_\ell^2 - \sum c_\ell A_\ell \hat{\rho}_\ell \sum c_\ell A_\ell A_\ell' = 0 \tag{3.12}$$

where all the sums are from $\ell = p+1$ to $\ell = N$.

The procedure for extracting the shock location, jump magnitude, and smooth part of the solution from the raw data $\rho(x,y_j)$ (given in Table 2) is exactly the same as described above for the Fourier problem.

For the wedge flow problem considered here, this procedure applied in the case of a coarse net $(N = 8)$ located the shock with an error only in the fourth significant figure. The smooth part was recovered to within 1% at the worst field point.

4. CONCLUSIONS

A procedure has been developed for extracting the piecewise smooth solution of hyperbolic system from the raw oscillatory data obtained by pseudospectral methods. The theory, so far, is precise only for the linear case with constant coefficients. Plausibility arguments point out that this procedure will also work for nonlinear operators when steady state has been achieved. The numerical results presented herein bear out this conjecture.

It remains to try and understand the phenomenon better in the nonlinear cases where waves are propagated with different speeds at different field points and at different times.

5. REFERENCES

[1] Gottlieb, D. and Orszag, S.A., "Numerical Analysis of Spectral Methods: Theory and Applications," CBMS Regional Conference Series in Applied Mathematics, 26, SIAM, 1977.

[2] Shannon, C.E., "A Mathematical Theory of Communication," Bell System Technical Journal, 27, pp. 379 - 423, 623 - 658, 1948.

[3] Lax, P.D., "Accuracy and Resolution in the Computations of Solutions of Linear and Nonlinear Equations" in Recent Advances in Dimensional Analysis, MRC University at Wisconsin, Academic Press, 1978, pp. 107 - 117.

[4] Gottlieb, D., "Spectral Methods for Compressible Flow Problems," ICASE Report No. 84-29, To appear in the Proceedings of the ICYNAFD.

1. Report No. NASA CR-172558 ICASE Report No. 85-14	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle INFORMATION CONTENT IN SPECTRAL CALCULATIONS		5. Report Date February 1985 6. Performing Organization Code
7. Author(s) Saul Abarbanel and David Gottlieb		8. Performing Organization Report No. 85-14 10. Work Unit No.
9. Performing Organization Name and Address Institute for Computer Applications in Science and Engineering Mail Stop 132C, NASA Langley Research Center Hampton, VA 23665		11. Contract or Grant No. NAS1-17070, 13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code 505-31-83-01

15. Supplementary Notes

Additional Support: AFOSR Contract No. AFOSR 83-0089.
Langley Technical Monitor: J. C. South, Jr.
Final Report

16. Abstract

Spectral solutions of hyperbolic partial differential equations induce a Gibbs phenomenon near local discontinuities or strong gradients.

A procedure is presented for extracting the piecewise smooth behavior of the solution out of the oscillatory numerical solution data. The procedure is developed from the theory of linear partial differential equations. Its application to a non-linear system (the two-dimensional Euler equations of gas dynamics) is shown to be efficacious for the particular situation.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
spectral methods Gibbs phenomenon information content hyperbolic systems post-processing	64 - Numerical Analysis Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	14	A02

For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1985